A unified formulation of the spectra of temperature fluctuations in isotropic turbulence

1991 J. Phys. A: Math. Gen. 24 4721

(http://iopscience.iop.org/0305-4470/24/20/007)

J. Phys. A: Math. Gen. 24 (1991) 4721–4729. Printed in the UK

A unified formulation of the spectra of temperature fluctuations in isotropic turbulence

Bhimsen K Shivamoggi and Larry C Andrews
University of Central Florida, Orlando, FL 32816, USA

Received 5 April 1991, in final form 17 June 1991

Abstract. This paper gives a unified formulation of the spectra of temperature fluctuations in isotropic turbulence in *all* subregimes of wavenumbers. This is accomplished by making a systematic analysis of the effects of the fluid convection, viscosity and thermal diffusivity on the temperature spectrum. A generalized Heisenberg–von Weizsacker type model is used to describe the statistical interaction of the Fourier components of velocity and temperature. The latter is described in terms of kinetic energy and temperature variance transfer coefficients given by moments of the two spectra for large-wavenumber modes.

1. Introduction

The temperature field is made random by the irregular movements of the fluid and acquires statistical properties which are directly related to those of the turbulent velocity field. A prominent effect of the turbulent motion on the temperature field is a continual reduction of the length scale of the temperature fluctuations. The random convection of material elements of the fluid leads to distortion of these elements, and in the absence of thermal diffusion, a statistical increase in the gradients of temperature. The continual increase in the magnitude of temperature gradients due to random convection will ultimately be halted by the smoothing action of thermal diffusion, and no further refinement of the temperature distribution can occur; this determines a length scale characterizing the smallest temperature 'eddies.' Arguments similar to those used in Kolmogorov's (1941) theory may then be invoked to surmise that the small-scale structure of the temperature field has a measure of universality and has statistical properties which depend only weakly on the large-scale features of the field. The small-scale statistics of a scalar like temperature diffusing in a homogeneous turbulence have been studied by assuming that the temperature fluctuations are also homogeneous. Homogeneous turbulence is a theoretical idealization which implies that statistical quantities do not depend upon their absolute position in space. This simplifies the treatment of turbulence by separating the interaction of the turbulent fluctuations with themselves from their interaction with the mean flow. Even though homogeneous turbulence has not been found either in the laboratory or in nature, it nonetheless provides a useful approximation to small-scale fluctuations in a convected scalar-like temperature which has a characteristic length scale small compared with the scale of inhomogeneity. Such a situation may arise, for instance, in the case of a grid turbulence where a slight statistically homogeneous heating of the fluid close to the grid will produce a fluctuating temperature field which will be homogenized downstream of the grid by the dissipative action of the thermal-diffusive effects.

0305-4470/91/204721+09$03.50

The problem to be studied is as follows (Batchelor 1959). The temperature field $\theta(x, t)$ in the fluid is governed by the equation

$$\frac{\partial \theta}{\partial t}+v \cdot \nabla \theta=\kappa \nabla^2 \theta \tag{1}$$

where v is the velocity of the fluid and is usually assumed to be independent of θ (i.e., the temperature is considered to be a dynamically passive scalar). One assumes (as implied by equation (1)), that there are only two properties of temperature that are relevant to the present mechanical processes. One is the property of constancy of temperature of a fluid particle in the absence of thermal diffusivity; the other is that the temperature is subject to thermal diffusion characterized by the thermal diffusivity κ. The fluid, taken as incompressible, is in turbulent motion at high Reynolds number, and the length-scale characterizing the energy-containing eddies is L. The problem is to determine the statistical properties like the wavenumber spectrum of those components of the spatial distribution θ that have a length scale compared with L.

The first work on this problem was that due to Obukhov (1949) and Corrsin (1951), who looked into the primary features of the effect of random convection on the spatial distribution of θ and pointed out that the quadratic term in equation (1) represents a transfer from Fourier components of θ-distribution of low wavenumber to those at high wavenumber similar to the case with the turbulent velocity field. Therefore, they argued that the hypotheses of Kolmogorov universal equilibrium theory for the v-distribution apply equally well to the θ-distribution, and if the Reynolds number and the Peclet number of the turbulence are sufficiently high, the statistical properties of the small-scale components of the θ-distribution in a group called the inertial-convective subrange are homogeneous, isotropic and steady, irrespective of the detailed form of the properties of the components with length scale of order of L. Moreover, the statistical properties of the small-scale components are affected by these large-scale components only inasmuch as the latter determine the magnitude of the rate of transfer of the scalar variance from large-scale to small-scale components. The θ-spectrum in the inertial-convective range is assumed to depend only on the average rate per unit volume of the fluid at which the scalar variance is transferred in the spectral space χ and the average rate of viscous dissipation of kinetic energy per unit mass of the fluid ε.

The first step in a systematic generalization of the Obukhov-Corrsin model to include the effects of viscosity ν and thermal diffusivity κ is the classification of the various subranges of wavenumbers depending on the relative values of ν and κ. If, for instance, $\kappa \gg \nu$, then the temperature spectrum will experience diffsuvie effects at wavenumbers low enough for the kinetic energy spectrum not to be influenced by the viscous effects. The terminology for the various wavenumber ranges may be based on whether the wavenumber lies in the inertial or viscous range of range of the energy spectrum and the convective or diffusive range of the temperature spectrum. Thus, one defines the following sets of wavenumber ranges depending on whether the Prandtl number Pr is much smaller or much larger than unity:

$\mathrm{Pr} \ll 1$: inertial-convective: $L^{-1} \ll k \ll (\varepsilon/\kappa^3)^{1/4}$

inertial-diffusive: $(\varepsilon/\kappa^3)^{1/4} \ll k \ll (\varepsilon/\nu^3)^{1/4}$

$\mathrm{Pr} \gg 1$:	inertial–convective:	$L^{-1} \ll k \ll (\varepsilon/\nu^3)^{1/4}$
	viscous–convective:	$(\varepsilon/\nu^3)^{1/4} \ll k \ll (\varepsilon/\kappa^3)^{1/4}$
	viscous–diffusive:	$k \gg (\varepsilon/\kappa^3)^{1/4}$

the viscous–diffusive range is usually not recognized in cases for which $\mathrm{Pr} \ll 1$.

The Obukhov–Corrsin theory does not take into account the intermittency in the flow that arises due to the non-Gaussian nature of the small-scale statistics and leads to the spatial randomness of dissipation rates χ and ε, which would be expected to depend on Reynolds and Peclet numbers and to cause at the lower end of the convective subrange systematic departures from the Obukhov–Corrsin scaling laws which use mean dissipation values. One could now follow Obukhov (1962) and Kolmogorov (1962) and reformulate the ideas in the Obukhov–Corrsin theory by introducing local mean dissipations, determined by averaging over a suitably small region in space. One could then assume that these are random variables following logarithmic normal distributions, and deduce how some of the original local similarity arguments applied to structure functions would change under the new interpretation (Shivamoggi *et al.* 1989). (A theory based on the joint probability distribution of the local mean dissipations as a bivariate log-normal was given by Van Atta (1971) and with some further experimental corroboration by Antonia and Van Atta (1975). But this theory is restricted to Prandtl members of unity so that it cannot treat the viscous–convective and inertial-diffusive regimes prevalent for Prandtl numbers away from unity.) Alternatively, one could follow Mandelbrot (1976) and argue that the deviations from the Obukhov–Corrsin scaling laws are related to the fractal aspects of the geometry of turbulence (Chechetkin *et al* 1990). In particular, one may assume that the average dissipation rates χ and ε are concentrated on sets with non-integer Hausdorff dimensions. One may formulate these ideas in a simpler way through the so-called β-model (Frisch *et al* 1978). The key assumption in this model is that the fluxes of energy and scalar variance are transferred only to fixed fractions β and γ of the eddies downstream in the cascade. However, the β-model has been found not to fit the experimental measurements (Antonia *et al* 1984) well, because a single parameter given by the uniform fractal dimension is not adequate to describe the fractal nature of turbulence. A third approach is to adopt the joint gamma distribution for the average dissipation rates χ and ε (Andrews and Shivamoggi 1990). This model was a natural consequence of using the marginal gamma distribution to describe the local kinetic energy dissipation ε by Andrews *et al* (1989), and is found to fit the experimental measurements (Antonia *et al* 1984) well.

The intermittency corrections mentioned in the foregoing may, however, be too small to allow an experimental verification at the usual level of resolution of the energy and temperature spectra. One may therefore seek to make a systematic analysis of the effect of fluid convection, viscosity and thermal conductivity on the temperature spectrum. However, this approach has proved to be controversial since conflicting results have been given by several workers. The first work along this line was that of Batchelor *et al* (1959) who deduced that the temperature goes like $k^{-17/3}$ in the inertial–diffusive range. This result was also deduced by Herring *et al* (1982) using a quasinormal approximation and by Kraichnan (1968) using the Lagrangian-history direct-interaction approximation. Recently Chasnov *et al* (1988, 1989) also confirmed this result using a numerical simulation but the precise range of wavenumbers for which this result holds was not clear. However, Gibson (1968a, b) presented cases showing a power law of k^{-3} for this range, and Corrsin (1964) and Leith (1968)c predicted an exponential form for the spectrum. One would have hoped that this controversy should be resolved experimentally. But experiments have not been possible in this regime. We propose to try to clarify the existing picture by using instead the generalized Heisenberg-von Weizsacker type model for the statistical interaction of the Fourier components of $\boldsymbol{v}$ and θ.

According to the generalized Heisenberg-von Weizsacker type model, the transfer of the scalar variance from large to small wavelengths is described by a gradient-diffusion type cascade process (i.e. a small-scale rapidly adjusting motion superimposed on a large-scale slowly adjusting motion) characterized by a 'eddy thermal conductivity' produced by large wavenumber modes acting to remove scalar variance from small wavenumber modes. This idea is similar to the one originally proposed by Heisenberg (1948) and von Weizsacker (1948) for the transfer of turbulent kinetic energy and generalized by von Karman (1948). The latter model has been shown (Uberoi and Narain 1974) to compare more favourably with experiments than the original Heisenberg-von Weizsacker model. We will show in this paper that the generalized Heisenberg-von Weizsacker type model gives the temperature spectra in *all* the subregimes of wavenumbers (namely, the inertial-convective, viscous-convective, inertial-diffusive and viscous-diffusive regions) in a unified way through a single formula. (It may be mentioned that an eddy thermal diffusivity model has also been recently given by Charnov *et al* (1989), but this model prertains only only to the inertial-diffusive range and lacks the broad scope and generality of the present formulation.) We will demonstrate the existence of a bump in the temperature spectrum in the viscous-convective regime observed in experiments (Williams and Paulson 1977, Champagne *et al* 1977). The bump has also been predicted by the eddy-damped quasinormal Markovian model (Herring *et al* 1982) and the recent numerical simulation of Kerr (1990). The bump in the termperature spectrum therefore has a sound physical basis even though in an experimental situation some of it may be due to the breakdown of Taylor's hypothesis because of the changing mean flow conditions.

2. Fourier analysis of the turbulent fields

Fourier analysis of the velocity and temperature fields, when they are stationary random functions of position, affords a view of the turbulent motion as composed of the superposition of motions of a large number of components of different scales. These component contribute additively to the total kinetic energy and total scalar variance and interact with each other according to the inertial and convective terms in the governing equations. Fourier representation is well justified for infinitely extended homogeneous random fields, like the ones we have under discussion.

Let us express the flow properties at any point $\boldsymbol{x}$ at time t, as a superposition of plane waves of the form

$$\boldsymbol{v}(\boldsymbol{x}, t) = \sum_{\boldsymbol{k}} \boldsymbol{V}(\boldsymbol{k})\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \qquad \theta(\boldsymbol{x}, t) = \sum_{\boldsymbol{k}} \Theta(\boldsymbol{k})\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}. \tag{2}$$

We have dropped the argument t, on the right-hand side in (2), for convenience. We then obtain from equation (1)

$$\left(\frac{\partial}{\partial t} + \kappa k^2\right)\Theta(\boldsymbol{k}) = -\mathrm{i}k_m \sum_{\boldsymbol{k}'} V_m(\boldsymbol{k}')\Theta(\boldsymbol{k}-\boldsymbol{k}'). \tag{3}$$

3. Generalized von Karman–Heisenberg–von Weizsacker-type model for the excitation of temperature fluctuations

We have from equation (3) the following equation for the isotropic temperature spectrum function $\Gamma(k)$:

$$\left(\frac{\partial}{\partial t}+2\kappa k^2\right)\Gamma(k)=\sum_{\boldsymbol{k}'} U_m(\boldsymbol{k}, \boldsymbol{k}') \tag{4}$$

where

$$\Gamma(k)\equiv\tfrac{1}{2}|\Theta(\boldsymbol{k})|^2 \tag{5}$$

and

$$U_m(\boldsymbol{k}, \boldsymbol{k}')=-\mathrm{i}k_m\Theta(\boldsymbol{k})V_m(\boldsymbol{k}')\Theta(\boldsymbol{k}-\boldsymbol{k}'). \tag{6}$$

When the volume of the flow region becomes large, we may replace the Fourier sum in (4) by a Fourier integral

$$\sum_{\boldsymbol{k}'} U_m(\boldsymbol{k}, \boldsymbol{k}')=\frac{1}{8\pi^3}\int Q(\boldsymbol{k}, \boldsymbol{k}')\,\mathrm{d}\boldsymbol{k}' \tag{7}$$

where $Q(\boldsymbol{k}, \boldsymbol{k}')$ is the net gain of scalar variance by modes of wavenumber $\boldsymbol{k}$ from all modes in the range $\boldsymbol{k}'$ to $\boldsymbol{k}'+\mathrm{d}\boldsymbol{k}'$. In order to write an expression for this quantity, it is necessary to make some assumption about the convective transfer of scalar variance across the spectrum.

If each mode in the range of wavenumbers from $k'=k$ to $k'=\infty$ is to make a separate and similar contribution to the 'eddy thermal diffusivity' $\tilde{\kappa}(k)$, then by dimensional considerations we may write

$$Q(\boldsymbol{k}, \boldsymbol{k}')=\begin{cases}2A[E(k')]^{3/2-n}k'^{1/2-m}[\Gamma(k)]^n k^m & k'<k\\ -2A[\Gamma(k)]^{3/2-n}k^{1/2-m}[E(k')]^n k'^m & k'>k\end{cases} \tag{8}$$

where A is a universal constant and m and n are arbitrary constants. Equation (8) implies that the process of transfer of scalar variance from large to small wavelengths is described by a gradient-diffusion type cascade (as discussed in section 1) and that the eddy thermal diffusivity $\tilde{\kappa}(k)$ depends on the kinetic energy density $E(k)$ and the wavenumber k only (see (10) below) which is reasonable because the temperature field has been assumed to behave like a passive scalar made random upon advection by the turbulent velocity field. This idea has its antecedents in the earlier works of Taylor (1922) and Saffman (1969) on the diffusion of a passive scalar in a turbulent velocity field wherein the diffusion coefficient is expressed in terms of the velocity correlation function.

The rate of loss of scalar variance by modes with wavenumbers less than some values k is given by

$$\int_0^k \frac{\partial\Gamma(k'')}{\partial t}\,\mathrm{d}k''=-2\kappa\int_0^k \Gamma(k'')k''^2\,\mathrm{d}k''-2\tilde{\kappa}(k)\int_0^k [\Gamma(k'')]^{3/2-n}k''^{1/2-m}\,\mathrm{d}k'' \tag{9}$$

where

$$\tilde{\kappa}(k)\equiv A\int_k^\infty [E(k')]nk'^m\,\mathrm{d}k'. \tag{10}$$

Let us now replace the left-hand side in equation (9) by the total rate of decay of scalar variance, χ. (This is valid for values of k sufficiently large so that

$$\int_0^k \Gamma(k'')\,\mathrm{d}k'' \gg \int_k^\infty \Gamma(k'')\,\mathrm{d}k''$$

which, in turn, is valid if only a negligible amount of scalar variance is contained in wavenumbers greater than k.) One then obtains from equation (9)

$$2\kappa\Gamma(k)k^2+[2A\{E(k)\}^n k^m]\frac{[-\chi+2\kappa\int_0^k \Gamma(k'')k''^2\,\mathrm{d}k'']}{2\tilde{\kappa}(k)}+2\tilde{\kappa}(k)[\Gamma(k)]^{3/2-n}k^{1/2-m}=0. \qquad (11)$$

Equation (11) is the main result of the present paper; the corresponding result for the velocity field was given by von Karman (1948). A solution of equation (11), for arbitrary values of m and n has not been obtained. However, it is possible to obtain the forms of solution of equation (11) in the limit of small and large values of the wavenumber k.

For the inertial-convective range, which corresponds to $\kappa \ll \tilde{\kappa}(k)$ and $\nu \ll \tilde{\nu}(k)$, noting that now $E(k)\sim k^{-5/3}$ (Obukhov 1949, Corrsin 1951), (11) gives the well known result:

$$\Gamma(k)\sim k^{-5/3}. \qquad (12)$$

For the viscous-convective range, which corresponds to $\kappa \ll \tilde{\kappa}(k)$ and $\nu \gg \tilde{\nu}(k)$, noting that now $E(k)\sim k^{-(m-2)/(n-1)}$ (see Uberoi and Narain 1974), on writing (11) in the following form,

$$2\kappa\int_k^\infty \Gamma(k'')k''^2\,\mathrm{d}k''=2A\int_k^\infty [E(k')]^n k'^m\,\mathrm{d}k'\int_0^k [\Gamma(k'')]^{3/2-n}k''^{1/2-m}\,\mathrm{d}k'' \qquad (13)$$

we obtain

$$\Gamma(k)\sim k^{-\frac{5}{2}-[(m-2)/(n-1)]n/\frac{3}{2}-n}. \qquad (14)$$

This gives Batchelor's (1959) results, $\Gamma(k)\sim k^{-1}$, for

(i) $n<1$ and $m<\frac{1}{3}(5n+1)$, or

(ii) $1<n<\frac{3}{2}$ and $m>\frac{1}{3}(5n+1)$

so that $E(k)\sim k^{-p}, p>\frac{5}{3}$ as befits a viscous regime. The rise in the temperature spectrum indicated by the above result describes the bump in the temperature spectrum observed by Williams and Paulson (1977) and Champagne *et al* (1977). Physically, this bump is due to the fact that the viscous dissipation of the 'driver' velocity fluctuations causes the 'driven' temperature fluctuations to pile up in the wavenumber space.

For the inertial-diffusive range, which corresponds to $\kappa \gg \tilde{\kappa}(k)$ and $\nu \gg \tilde{\nu}(k)$, noting that now $E(k)\sim k^{-5/3}$, (13) gives

$$\Gamma(k)\sim k^{-\frac{5}{3}n+m-2} \qquad (15)$$

which yields the result of Batchelor *et al* (1959), $\Gamma(k)\sim k^{-17/3}$, for $m=-2$ and $n=1$. The steeper temperature spectrum indicated by the above result is due to the fact that the temperature fluctuations are being rushed to higher wavenumbers by the strong inviscid velocity fluctuations while being dissipated at the same time by thermal diffusivity. The verification of the spectral laws in the inertial-diffusion regime has been tenuous. This is due in part to the exotic nature of the low Prandtl-number materials like liquid mercury and sodium and the concomitant experimental difficulties have yet to be adequately resolved.

For the viscous-diffusive range, which corresponds to $\kappa \gg \tilde{\kappa}(k)$ and $\nu \gg \tilde{\nu}(k)$, noting that now $E(k) \sim k^{-(m-2)/(n-1)}$, (13) gives

$$\Gamma(k) \sim k^{-(m-2)/(n-1)}. \tag{16}$$

Equation (16) is identical to the spectral law of $E(k)$ in the viscous regime, as it should be, because, under the assumption of temperature behaving like a passive scalar, the spectral properties of the temperature field must be the same as those of the velocity field in the common dissipative regimes of the two fields. Thanks to the presence of two free parameters m and n, (16) can give an arbitrarily steep power law.

The above results regarding the shape of the θ-spectrum in the various wavenumber ranges are compared with the previous results in table 1.

Table 1. Comparison of previous and present results.

Subrange	Previous results	Presented results
Inertial-convective	$k^{-5/3}$ (Obukhov, Corrsin)	$k^{-5/3}$
Viscous-convective	k^{-1} (Batchelor)	$k^{-\frac{3}{2}-[(m-2)/(n-1)]n/\frac{3}{2}-n}$ m, n free parameters
Inertial-diffusive	$k^{-17/3}$ (Batchelor *et al*) k^{-3} (Gibson) $k^{-5/3}\,\mathrm{e}^{-\frac{3}{2}\kappa\varepsilon^{-1/3}k^{4/3}}$ (Corrsin)	$k^{-\frac{5}{3}n+m-2}$ m, n free parameters
Viscous-diffusive	$k^{-1}\,\mathrm{e}^{\kappa\gamma^{-1}k^2}$ (Batchelor)	$k^{-(m-2)/(n-1)}$ m, n free parameters

4. Heisenberg-von Weizsacker type model for the excitation of temperature fluctuations

For the Heisenberg-von Weizsacker type model for the convective transfer of scalar variance, for which $m = -\frac{3}{2}$ and $n = \frac{1}{2}$, an explicit solution for equation (11) can be given:

$$\Gamma(k) = \frac{A\chi\{E(k)\}^{1/2}k^{-7/2}}{2\{\kappa + \tilde{\kappa}(k)\}^2} \tag{17}$$

where

$$\tilde{\kappa}(k) \equiv A\int_k^\infty \sqrt{\frac{E(k')}{k'^3}}\,\mathrm{d}k'. \tag{18}$$

Equation (18) leads to the following results for the various ranges:

$$\text{viscous-convective:} \qquad \Gamma(k) \sim k \tag{19}$$

$$\text{inertial-diffusive:} \qquad \Gamma(k) \sim k^{-13/3} \tag{20}$$

$$\text{viscous-diffusive:} \qquad \Gamma(k) \sim k^{-7}. \tag{21}$$

It is to be noted that (20) agrees with the result given by Ogura (1958). Equation (21) is identical to the well known spectral law for $E(k)$ in the viscous regime with the Heisenberg-von Weizsacker model. On the other hand, interestingly enough, (19) can be recovered by a model based on a stationary continuous spectral cascading process (due to the advective term in equation (1)) for the transfer of the scalar variance to large wavenumbers. This idea is similar to the one originally proposed by Onsager (1949) in the form of a discrete cascade and generalized to the form of a continuous cascade by Corrsin (1964) and Pao (1965) for the transfer of turbulent kinetic energy at large wavenumbers.

If $S(k)$ is the spectral flux function of the Θ-field, i.e. it is the rate at which the spectral content of the Θ-field flows in wavenumber space from wavenumbers smaller than k to wavenumbers greater than k, we have

$$\frac{\mathrm{d}S(k)}{\mathrm{d}t}=\left\{\begin{array}{l}\text{rate of gain or loss of spectral}\\ \text{content of } \overline{\theta^2} \text{ per unit wavenumber}\end{array}\right\} \tag{22}$$

which is a generalization of Onsager's proposition to a non-conservative cascade process. A non-conseervative cascade is one in which a steady temperature spectrum can be maintained against the dissipative action of thermal diffusitivity κ only by a net gain of the scalar variance resulting from the interaction of the Fourier components of temperature and velocity.

Now, in the viscous-convective range, the thermal-diffusive effects are not important, so that we have a conservative cascade process

$$\frac{\mathrm{d}S(k)}{\mathrm{d}k}=0. \tag{23}$$

If we now visualize the transfer of scalar variance as a cascading process in which the spectral content of the scalar variance is continuously transferred to ever larger wavenumbers, the flux of the scalar variance across k can then be written as

$$S(k)=\Gamma(k)\frac{\mathrm{d}k}{\mathrm{d}t} \tag{24}$$

where $\mathrm{d}k/\mathrm{d}t$ is the spectral cascading rate. Let us now assume that this process depends on ε (the rate at which the turbulent kinetic energy is fed to small eddies), on the viscosity ν, and on the wavenumber k (or equivalently, the size of the small eddies). On dimensional grounds, we have then for the spectral cascading rate

$$\frac{\mathrm{d}k}{\mathrm{d}t}\sim\varepsilon\nu^{-2}k^{-1}. \tag{25}$$

Using (24) and (25), equation (23) then gives

$$\Gamma(k)\sim k \tag{26}$$

in agreement with (19).

5. Discussion

We have given in this paper a unified formulation of the spectra of temperature fluctuations in isotropic turbulence in *all* subregimes of wavenumbers. For this purpose, a generalized Heinsenberg–von Weizsacker type model has been used to describe the statistical interaction of the Fourier components of velocity and temperature. This model is able to describe in a unified way not only the well known result for the inertial-convective regime but, by adjusting its free parameters, also the results for other subregimes of wavenumbers in a satisfactory way. The results compare favourably with other existing formulations of the spectra of temperature fluctuations which give only a piece-by-piece treatment of the various subregimes of wavenumbers.

References

Andrews L C, Phillips R L, Shivamoggi B K, Beck J K and Joshi M 1989 *Phys. Fluids* A 1 999
Andrews L C and Shivamoggi B K 1980 *Phys. Fluids* A **2** 105
Antonia R A, Hopfinger E J, Gagne Y and Anselmet F 1984 *Phys. Rev.* A **30** 2704
Antonia R A and Van Atta C W 1975 *J. Fluid Mech.* **67** 273
Batchelor G K 1959 *J. Fluid Mech.* **5** 113
Batchelor G K, Howells I D and Townsend A A 1959 *J. Fluid Mech.* **5** 134
Champagne F H, Friehe C A, LaRue J C and Wyngaard J C 1977 *J. Atmos. Sci.* **34** 515
Chasnov J, Canuto V M and Rogallo R S 1988 *Phys. Fluids* **31** 2065
—— 1989 *Phys. Fluids* A **1** 1698
Chechetkin V R, Lutsvinov V S and Turygin A Yu 1990 *J. Stat. Phys.* **61** 589
Corrsin S 1951 *J. Appl. Phys.* **22** 469
—— 1964 *Phys. Fluids* **7** 1156
Frisch U, Sulem P L and Nelkin M 1978 *J. Fluid Mech.* **87** 719
Gibson C H 1968a *Phys. Fluids* **11** 2305
—— 1968b *Phys. Fluids* **11** 2316
Heisenberg W 1948 *Z. Phys.* **124** 628
Herring J, Schertzer D, Lesieur M, Newman G, Chollet J and Larcheveque M 1982 *J. Fluid Mech.* **124** 411
Kerr R M 1990 *J. Fluid Mech.* **211** 309
Kolmogorov A N 1941 *Dokl. Akad. Nauk SSSR* **30** 301
—— 1962 *J. Fluid Mech.* **13** 82
Kraichnan R H 1968 *Phys. Fluids* **11** 945
Leith C E 1968 *Phys. Fluids* 11 1612
Mandelbrot B 1976 *Lecture Notes in Mathematics* vol 565 (Berlin: Springer) p 121
Obukhov A M 1949 *Izv. Akad. Nauk SSSR, Ser. Geogr. i Geofiz.* **13** 58
—— 1962 *J. Fluid Mech.* **13** 77
Ogura Y 1958 *J. Meteorol* **15** 539
Onsager L 1949 *Nuovo Cimento, Suppl.* **6** 279
Pao Y H 1965 *Phys. Fluids* **8** 1063
Saffman P G 1969 *Phys. Fluids* **12** 1786
Shivamoggi B K, Phillips R L and Andrews L C 1989 *J. Phys. A: Math. Gen.* **22** 1253
Taylor G I 1922 *Proc. London Math. Soc. Ser. 2* **20** 196
Uberoi M S and Narain J P 1974 *Z. Phys.* **269** 1
Van Atta C W 1971 *Phys. Fluids* **14** 1803
von Karman T 1948 *Proc. Natl. Acad. Sci., USA* **34** 530
von Weizsacker C F 1948 *Z. Phys.* **124** 614
Williams R M and Paulson C A 1977 *J. Fluid Mech.* **83** 547